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Assignment 7: MTH 213, Fall 2017

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QUESTION 1. a) Let $A = \{1, 3, 4\}$. Define "$\leq$ " on A :$\forall a, b \in A$, we say $a \leq b$ iff $a < b^3$. Then "$\leq$" is not a partial order relation on A. Why?

b) Let $A = \{-5, -4, -2, -1\}$. Define "$\leq$" on A :$\forall a, b \in A$, we say $a \leq b$ iff $a^2 > b^3$. Then "$\leq$" is not a partial order relation on A. Why?

Is $\leq$ an equivalence relation? If yes, how many elements does "$\leq$" have? Do not write them down!

c) Let $A = \{2, 4, 6, 8, 10, 12, 18\}$ and $B = \{1, 2, 4\}$. Define "$\leq$ " on A :$\forall a, b \in A$, we say $a \leq b$ iff $b = ma$ for some $m \in B$. Then "$\leq$" is a partial order relation on A. Why? Find maximal, minimal, least, and greatest elements of A under $\leq$ if they exist.

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Assignment 8

a) Symmetry $a \leq a$ iff $a < a^3$ $\forall a \in A$

so for element 1 $\Rightarrow$ $1 < 1^3$ not true

Not a partial order relation

b) 1) Symmetry, $a \leq a$ iff $a^2 > a^3$ $\forall a \in A$

$\therefore$ $\forall a \in A$, positive > negative so it is true

Anti-reflexive: Assume $a \leq b$ and $a \neq b$. Show that $b \leq a$

$\forall a, b \in A$, $a^2 > b^3$ but also $b^2 > a^3$

Example, $a = -5$, $b = -4$ $\Rightarrow$ $a^2 > b^3$ $\Rightarrow$ $25 > -64$

if $b = -5$, $a = -4$ $\Rightarrow$ $b^2 > a^3$ $\Rightarrow$ $25 > -64$

Not true since $a \leq b$ and $b \leq a$

ii) From above,

Symmetry is true

Note: name of the relation is <= and not =. So we must use the name <=

Reflexive: $a^2 > b^3$ and $b^2 > a^3$ so a<= b and b<= a so true

Transitive: Assume a<= b and b <= c.

a<=b $\Rightarrow$ $a^2 > b^3$ and b<=c $\Rightarrow$ $b^2 > c^3$

square is always positive and cube is always negative for the set A.

$\therefore$ $a^2 > c^3$ $\Rightarrow$ a <= c

"$\leq$" is an equivalence relation

Note that there is only one equivalence class since a \leq b for every a, b in A. Thus [-2] = A.
The elements of <= form a subset of AXA. Since we have one equivalence class , the elements of <= is just A X A. It is clear that |A X A| = 16 elements

c) Symmetry: Show that $a \leq a$

$a = ma$ if $m = 1$ so true

Anti-reflexive: Assume $a \neq b$ and $a \leq b$. Show that $b \not\leq a$.

if $b = ma$ then to get $a = mb$ then m has to be a fraction which does not exist. Hence $b \not\leq a$

Transitive: Note that the set here is finite. By staring we see only three different elements in A where a<= b and b <= c (namely a = 2, b = 4, c = 8). So a <=b and b<=c. Show that a<=c, i.e. show that 2<= 8. Since 8 = 2X4 (m here equals 4 and 4 is in B). So 2 <= 8

Maximal: 18, 12, 10, 8

Minimal: 2, 6, 10, 18

Greatest: does not exist

Least: does not exist

Note: $\forall a$, $10 \not\leq a$ and $a \not\leq 10$

$\forall a$, $18 \not\leq a$ and $a \not\leq 18$